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Article

Investigating the impact of argumentation on high school students' achievement in probability

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Abstract

This study aimed to investigate the impact of an argumentation instructional approach on high school students' understanding and performance in probability in Zimbabwe. A sample of 120 students was randomly assigned to experimental and control groups, with the experimental group receiving argumentation-based instruction while the control group followed traditional teaching methods. Both groups completed standardized pre-tests and post-tests, with results analysed using t-tests and ANCOVA. Findings indicated a significant improvement in the experimental group's performance. The study concluded that the argumentation approach positively influenced students' understanding and achievement in probability. However, limitations included the study's focus on a single school and a short intervention duration, suggesting future research should encompass multiple schools and longitudinal designs. The findings have implications for mathematics education policy and curriculum development in Zimbabwe and similar contexts.

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argumentation; instructional approach; probability; advanced level; critical thinking

Introduction

In the past decade, argumentation has emerged as an effective instructional approach in mathematics education (Toulmin, 1958; Kuhn, 2010; Osborne, 2010). Toulmin (1958) defines argumentation as the process of making claims, providing evidence, and justifying views. This approach has been emphasized as a critical competency for reasoning and communication (Kuhn, 2010; Osborne, 2010). Erduran and Jiménez-Aleixandre (2012) argue that the argumentation approach holds significant potential for mathematics learning. They posit that through argumentation, students can construct, critique, and refine their arguments about probability, which encourages deep engagement with probability concepts and nurture a more profound conceptual understanding.

Probability plays a significant role in students' critical analysis and decision-making processes. However, it presents challenges, including difficulties in understanding complex concepts, predicting outcomes under uncertainty, and justifying reasoning. These challenges align closely with the processes of argumentation. The topic of probability is often counterintuitive, leading to numerous problems for students. Additionally, misconceptions surrounding probability concepts create a rich field for argumentative discourse. In Zimbabwe, advanced-level students (high school students in their twelfth and thirteenth years of schooling)

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generally perform poorly on probability questions in high-stakes examinations and struggle to apply probability concepts to real-life situations (Batanero & Álvarez-Arroyo, 2023). Recent studies have sought to address this research gap by assisting students in grasping the concept of probability and applying it in various contexts (Batanero & Álvarez-Arroyo, 2023).

The theoretical foundation of the argumentation teaching method supports its effectiveness in teaching probability. Von Glasersfeld (1995) affirms that this approach is rooted in constructivism, which posits that learners actively construct knowledge rather than passively acquiring it from teachers. Engaging in mathematical arguments enables students to express their reasoning and formulate conjectures about others' ideas. This engagement can lead to a better understanding of probability concepts while simultaneously developing communication and problem-solving skills. According to Garfield and Ben-Zvi (2014), the teaching of probability often emphasizes procedural knowledge over conceptual understanding, which contributes to students' difficulties in solving routine probability problems. They note that the challenge of bridging the gap between theoretical knowledge and practical application persists even at the advanced-level of secondary education.

The current research aims to fill this gap by investigating the impact of argumentation-based teaching and learning practices on students' understanding of probability at the advanced-level in Zimbabwe. The focus on argumentation in this context seeks to create a learning environment that encourages students to articulate their thoughts, critique the reasoning of others, and collaboratively construct knowledge of probability. This study contributes to the broader discourse on pedagogies that enhance student learning by examining how argumentation interacts with the development and attainment of students' probabilistic understanding, as well as addressing issues within the Zimbabwean education system and their implications for mathematics education. This study demonstrates that argumentation can provide opportunities to enhance mathematical reasoning and problem-solving skills, with probability serving as the vehicle for developing argumentative skills.

Research Objectives

1. To determine the effectiveness of the argumentation teaching approach in improving advanced-level students' overall performance in probability.
2. To compare the impact of argumentation-based learning strategies with traditional teaching methods on students' ability to solve complex, real-world probability problems.
3. To assess whether the effectiveness of argumentation-based learning strategies in teaching probability varies based on students' initial performance levels.

Hypotheses

H1: Students taught probability using argumentation-based learning strategies will show significantly higher post-test scores compared to students taught using traditional methods.

H2: Students taught using argumentation-based interventions will perform significantly better on complex, real-world probability problems in the post-test compared to those taught using traditional methods.

H3: The effect of the argumentation-based intervention on probability performance will differ significantly between students taught using argumentation and those taught using traditional approaches.

This study tests these hypotheses by evaluating the effectiveness of the argumentation strategy in the teaching of probability. The results may enrich pedagogical practices in mathematics education, guide curriculum development, and inform teacher training in Zimbabwe and beyond.

Theoretical Framework

The theory of argumentation has evolved since the mid-20th century, with various scholars contributing different perspectives, ultimately leading to our current understanding of how arguments can be constructed, evaluated, and utilized in diverse contexts (Olbrechts-Tyteca, 1969; Toulmin, 1958; Yackel & Cobb, 1996). Stephen Toulmin's (1958) work in "The Uses of Argument" marked a turning point in modern studies of argumentation. Toulmin challenged the prevailing belief that all arguments must conform to formal logic, proposing an informal model of reasoning. His model, which comprises claims, grounds, warrant, backing, qualifier, and rebuttals, provides a framework for analysing arguments across various fields, emphasizing the importance of context-specific criteria in argument evaluation.

Around the same time, the revival of research on rhetoric was exemplified by the publication of Perelman and Olbrechts-Tyteca's (1969) "The New Rhetoric: A Treatise on Argumentation." The authors focused on the audience, asserting that the primary objective of argumentation is to gain adherence from the addressed audience. This theory highlighted the social and persuasive nature of argumentation, underscoring the necessity of adapting arguments to specific audiences. Building on these foundations, van Eemeren and Grootendorst (2004) developed the pragma-dialectical argumentation theory. In their work, "A Systematic Theory of Argumentation: The Pragma-Dialectical Approach," they conceptualize argumentation as a social and rational activity aimed at resolving differences of opinion. Their research provides a collection of rules for critical discussion that connects normative ideals with empirically traceable descriptive realities of argumentative practice.

In recent years, Zarefsky (2014) has applied argumentation theory to political discourse in "Political Argumentation in the United States," demonstrating how political arguments are constructed and function within American democracy, emphasizing the practical implications of argumentation studies in real-world settings. More recently, Mercier and Sperber (2017) proposed an evolutionary approach to reasoning and argumentation in their book "The Enigma of Reason." They argue that reasoning is fundamentally "argumentative, aimed at constructing and evaluating arguments to persuade others". This perspective sheds new light on human reasoning, suggesting that our cognitive abilities have evolved more toward social persuasion than individual truth-seeking.

In the mathematical domain, Krummheuer (2007) and Yackel and Cobb (1996) situate the theory of argumentation within social constructivism. They assert that engaging in educational arguments helps students manage conceptual understanding and enhance their reasoning capacities. Stylianides et al. (2016) suggest that probabilistic argumentation enables students to address misconceptions, develop conceptual understanding, and enhance the application of concepts in various contexts. This study posits that social constructivism and argumentation theories can be integrated to create a conducive social environment for learning probability. In such an environment, students can make their thinking explicit, challenge one another's ideas, and collaboratively arrive at a shared understanding of probabilistic concepts. The theoretical framework provides a lens through which the effectiveness of argumentation in improving probability learning can be interpreted.

Previous research has identified significant challenges in the teaching and learning of probability, including misconceptions about randomness and independence (Batanero & Sanchez, 2005), difficulties in interpreting and applying probability concepts to real-world situations (Garfield & Ahlgren, 1988), and limited exposure to probability concepts during early education stages. According to Batanero, Chernoff, Engel, Lee, and Sánchez (2016), these challenges persist even at the advanced-level, highlighting the need for innovative teaching approaches. Significant research on the use of argumentation in mathematics education has been conducted globally. In the United Kingdom, Inglis et al. (2007) demonstrated that argumentation enables students to deeply understand and retain mathematical concepts. In the United States, studies by Kuhn (2010) and Yackel and Cobb (1996) have explored the development of argumentation skills through training in various domains, including mathematics. Stylianides, Bieda, and Morselli (2016) found that students' engagement in argumentative discourse about probabilistic concepts helped them confront and overcome common errors in conditional probability.

While several studies on argumentation in mathematics education exist in Africa, few have specifically addressed probability teaching. In South Africa, Ogunniyi and Hewson (2008) studied the impact of argumentation-based instruction on pre-service science teachers' ability to construct and evaluate arguments. Koçoğlu and Kanadlı (2024) explored the use of argumentation in secondary school mathematics, while Mereku and Mereku (2015) conducted a similar study, both recommending a shift toward more interactive, learner-centered pedagogies that include teaching argumentation to enhance learners' reasoning in mathematics. Although research on teaching and learning argumentation in probability is relatively underdeveloped in Zimbabwe, existing studies address broader issues in mathematics education. Serin (2023) recommends the adaptation of new methodologies to enhance mathematics teaching in the Zimbabwean context, specifically addressing the challenges faced by advanced-level students in understanding probability concepts.

These studies highlight important themes and gaps in the teaching of probability through argumentation in African and Zimbabwean contexts. The literature on argumentation is predominantly derived from Western perspectives, indicating a need for research that considers the unique cultural and educational landscape of Zimbabwe and other African countries. While many studies focus on primary or lower secondary education, there is a pressing need for research directly related to the use of argumentation in teaching probability at the advanced-level. Nsengimana, Mugabo, Ozawa, and Nkundabakura (2021) illustrated the potential of learner-centered approaches but raised concerns about the challenges of implementation within the constraints of the local educational system, such as inadequate resources. These gaps underscore the potential value of research into the use of argumentation in teaching probability at the advanced-level in Zimbabwe, which could yield valuable insights for both theory and practice in mathematics education.

Method

Research Design

This study employs a quasi-experimental design with pre-test and post-test assessments for student evaluation, which Creswell and Creswell (2017) identify as suitable for educational research. The design allows for comparison between traditional teaching methods and a socially constructed argumentation-based approach.

Participants

The sample for this study consisted of 120 advanced-level students from a school in Zimbabwe. The students were in their twelfth and thirteenth years of schooling. These learners were selected to represent a diverse demographic, encompassing various cultural backgrounds and educational experiences. To ensure a fair and unbiased distribution of participants, random assignment was employed to categorize the students into two groups: an experimental group and a control group, each comprising 60 students. This method of random assignment helped in mitigating selection bias and ensured that the observed differences in outcomes were of the intervention rather than pre-existing differences among the participants (Ong-Dean et al., 2010; Shadish et al., 2011; Troyer, 2022). The experimental group received instruction that incorporated argumentation strategies in teaching probability, while the control group continued with traditional instructional methods.

Data Collection

Quantitative data for this study were collected using standardized pre-test and post-test assessments of knowledge and skills in probability. Data analysis involved t-tests and ANCOVA to compare group performances, as recommended by Field (2013), using SPSS with a significance level set at $\alpha = 0.05$. Five specific tests were conducted: independent samples t-tests on pre-test scores to ensure group comparability; paired samples t-tests to assess score changes within each group from pre-test to post-test; an independent samples t-test to compare post-test scores between groups; ANCOVA to compare post-test scores while controlling for pre-test scores; and calculations of effect sizes (Cohen's d and partial eta squared) to determine the magnitude of group differences. This comprehensive statistical approach aimed to provide a thorough analysis of the data and the effectiveness of the intervention. The post-test, parallel in structure and level of difficulty to the pre-test but with different specific questions, was administered one week after the completion of the eight-week intervention period.

Pre-intervention

A pre-test was administered to both the experimental and control groups prior to the intervention. The test consisted of 30 items assessing various aspects of probability concepts, including 15 multiple-choice items, 10 short-answer problems requiring calculations, and 5 extended response questions set within familiar contexts in which students presented solutions in the form of arguments. The pre-test aimed to profile students in terms of their theoretical understanding, problem-solving abilities involving probability concepts, and argumentation abilities. The pre-test was administered to both groups one week before the intervention.

The Intervention

The teaching approach employed in this study was the argumentation-based instructional approach, implemented over an eight-week period. During this time, the experimental group attended four probability lessons per week, each lasting 60 minutes. The intervention included activities designed to foster argumentative discourse and critical thinking within probability lessons. Students participated in organized arguments related to probability concepts, creating opportunities to articulate their reasoning and consider alternative perspectives. They learned to construct mathematical arguments, analyze them, and evaluate their validity. Collaborative activities involved groups of ten students solving various problems related to probability scenarios, followed by presentations of their solutions to the class, where they answered questions and defended their reasoning. The teacher employed Socratic questioning techniques to guide students through the reasoning behind key probability concepts while challenging their assumptions. Writing-based argumentation was also incorporated to deepen student understanding and encourage them to identify potential misconceptions by justifying their arguments regarding how and why solutions in probability were reached.

Peer review was another essential component of the intervention. Students were explicitly taught how to provide constructive feedback on the strength and validity of their peers' arguments and solutions. Structured peer review sessions allowed students to evaluate the arguments presented by classmates. Real-life examples of probability were introduced, prompting students to express opinions on the applicability and limitations of probabilistic models in related real-life situations during group discussions. Finally, metacognitive reflections were employed, encouraging students to reflect on their learning process at the end of each week and assess how their understanding had changed or deepened as a result of the argumentation process. An illustrative question used in this intervention aimed to help students understand conditional probability, demonstrating how Vygotsky's Zone of Proximal Development (ZPD) and scaffolding were utilized through argumentation involving a probability scenario.

The problem was formulated as follows:

Out of a group of 100 students, 60 drink coffee, 70 drink tea, and 40 drink both. What is the probability that a student drinks coffee given that they drink tea?

The argumentation process unfolded as follows:

Student A (initial argument):

"I think it's 40/70 because 40 students drink both, and we know that the student drinks tea, so it must be out of 70."

This initial argument reflects the student's current understanding, marking the lower bound of their ZPD. The teacher then provided scaffolding through guiding questions:

Teacher:

"Why is the denominator 70? What is conditional probability?"

This prompted Student A to reconsider their reasoning, moving them into their ZPD.

Student B (counter-argument):

"I do not agree. I believe we must use the total number of people who drink coffee. Shouldn't we use 60 somewhere in the problem?"

This peer interaction further extended the ZPD for both students as they integrated various pieces of information. The teacher then provided a graphic scaffold by drawing a Venn diagram on the board to represent the overlapping sets of coffee and tea drinkers. This visual representation helped students structure the given information.

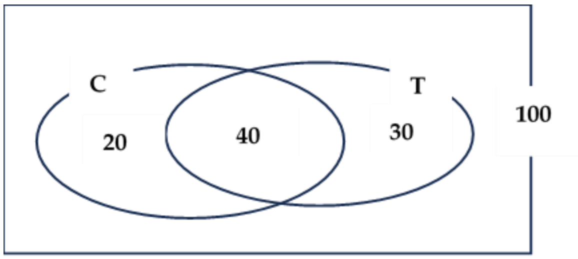

Figure 1. Venn diagram representing the problem

Student C (building on previous arguments):

'Looking at the Venn diagram, I see that out of the 70 tea drinkers, 40 also drink coffee. So, if we know someone drinks tea, the probability they also drink coffee is indeed 40/70.'

This contribution demonstrated how visual scaffolding and peer argumentation facilitated a more comprehensive understanding, advancing students through their ZPD.

The teacher then encouraged the class to formalize this reasoning using the conditional probability formula (Formula 1, below), gradually removing scaffolding as students demonstrated greater competence.

This example illustrates how argumentation, supported by appropriate scaffolding, enabled students to operate within their ZPD, leading to enhanced understanding of complex probability concepts. The process of articulating, defending, and refining arguments, coupled with teacher-provided scaffolds, facilitated students' progression from intuitive reasoning to more formal probabilistic thinking. This aligns with Vygotsky's theory, demonstrating how social interaction and guided support can foster cognitive development in the domain of probability. In contrast, the control group received traditional instruction in probability, primarily consisting of lecture-style teaching, individual problem-solving practice, and teacher-led discussions. In this group, the teacher provided learners with the appropriate conditional probability formula, which they used to calculate the required probability. The conditional probability formula given was:

Formula 1:

$P(C \mid T) = P(C \text{ and } T) / P(T)$

Where:

C = student drinks coffee

T = student drinks tea

Thus, the formula was expressed as:

P (coffee | tea) = P (student drinks coffee and tea) / P (student drinks tea). In this group, students substituted values into the formula to calculate the required probability.

A second contextual question was designed to assess students' critical thinking skills without the teacher's assistance as part of the argumentation evaluation for the experimental group. The teacher withdrew the Socratic questioning and scaffolding provided in the previous question to evaluate the extent of the argumentation's impact on students' reasoning. However, the control group was taught how to solve this problem without argumentation.

The question read:

A charity organization has set up a fundraising raffle. 1000 tickets are being sold at $5 each. The organization is offering a grand prize worth $25,000. The organization claims that buying more tickets significantly increases one's chances of winning and that it's a great opportunity to potentially win a dream car while supporting a good cause. Analyse this scenario, considering the following points:

1. Calculate and interpret the expected value of buying a single ticket.
2. Discuss how the number of tickets purchased affects the probability of winning and the expected value.
3. Evaluate the charity's claim about increasing chances of winning.
4. Consider any ethical implications of how the raffle is presented or organized.
5. Would you recommend buying tickets for this raffle? Justify your answer using probabilistic reasoning.

The results from the written responses revealed several themes regarding students' probabilistic reasoning and argumentation. The results are presented below according to these themes. Of the sixty students, 95% correctly calculated the expected value of buying a single ticket using the formula:

$$E(X) = 1/1000 * \$25{,}000 - \$5 = \$20.$$

However, interpretations of this outcome varied. On a positive note, seventy percent (n=42) reasoned that the positive expected value indicated that the raffle was a good investment. For example, one student argued: '*An expected value of $20 is positive, so in the long term, participating in this raffle mathematically has an advantage.*'

Conversely, twenty five percent (n=15) of the students justified their answers by highlighting the limitations of relying solely on expected value. One student commented: '*Though the expected value is positive, this is a one-time event. So, the law of large numbers doesn't apply. We therefore cannot rely only on the expected value for decision-making.*'

Different levels of analysis regarding the effect of purchasing multiple tickets were recorded, with seventy five percent (n=45) correctly arguing that the probability of winning increases linearly with the number of purchased tickets. The formula was stated as $P(win) = n/1000$, where n is the number of tickets purchased. Upon deriving this probability, one student asked during the discussions: '*Will the expected value remain the same then?*' In response, fifty percent of the students (n=30) explained in their arguments how the expected value changes with multiple tickets, proposing that the formula be expressed as:

$$E(X) = n/1000 * \$25{,}000 - n * \$5 = 25n - 5n = 20n.$$

For instance, after this analysis, thirty three percent (n=20) of students argued: *'The more tickets one buys, the greater the chances of winning, but also the greater the potential loss.'* This indicates that students' critical thinking had improved as they incorporated the concept of diminishing marginal returns. When considering the charity's claim that participants' chances would be 'significantly' improved, students demonstrated varying levels of critical thinking. Sixty percent (n=36) argued that the statement was correct but misleading. One student explained: *'Buying more tickets does increase the chances, but it is still very low even with multiple tickets. Therefore, it is misleading to use the word 'significantly' for people to understand.'*

Some students were concerned with ethical considerations, and seventy five percent of them found the claim to have potential for exploitation. Recommendations for purchasing raffle tickets varied; forty two percent of the students suggested buying tickets based on the positive expected value and the charity issue. Another thirty eight percent argued against purchasing tickets due to the low probability of winning and ethical concerns. Toulmin's argumentation pattern was used to assess the quality of the student's arguments, and it was found that seventy-two (n=43) of students provided explicit claims supported by data and warrants. Backings for warrants were included by fifty eight percent (n=35), demonstrating deeper engagement with probability concepts, while forty-five percent (n=27) used qualifiers effectively, showing the students' awareness of the limitations associated with their arguments. Those who included rebuttals were few and constituted thirty three percent (n=20), indicating an area needing improvement to develop well-rounded arguments. Throughout the intervention, instruction in both groups was carefully executed to ensure that the content covered aligned with the Zimbabwean Advanced-level Mathematics syllabus. This adherence ensured that research ethics were observed, as neither group was deprived of the essential examination requisites.

Post-intervention

A post-test was administered to the experimental and control groups after the intervention. The test consisted of 30 items assessing various aspects of probability concepts, including fifteen multiple-choice items, ten short-answer problems requiring calculations, and five extended response questions set within familiar contexts. The post-test aimed to profile students in terms of their theoretical understanding, problem-solving abilities involving probability concepts, and argumentation abilities following the intervention. The test items were randomized to minimize testing effects. Although the numerical values of each question differed, the level of difficulty remained consistent with the pre-test.

Reliability and Validity

The tests were designed to align with the Zimbabwe School Examination Council (ZIMSEC) A-Level Mathematics syllabus with the assistance of experienced statistics teachers. A pilot study involving 30 students was conducted to assess the clarity and difficulty level of the questions. Content validity was ensured through descriptions obtained for test items from three independent advanced-level mathematics teachers. An internal consistency reliability of the test ($\alpha > 0.80$) was found, indicating good reliability. The tests were administered under standardized conditions, lasting 90 minutes. Two independent ratings of the tests were conducted using a standardized rubric, with discrepancies resolved through discussion to ensure inter-rater reliability. This quantitative data collection approach provided a comprehensive and objective outline of students' probability knowledge and critical thinking skills before and after the intervention, allowing for inferences regarding the effectiveness of the argumentation intervention.

Data Analysis

The quantitative data analysis utilized t-tests and ANCOVA to compare group performances, following the recommendations of Field (2013). The analysis was conducted using SPSS, with a significance level set at $\alpha = 0.05$. Five specific statistical tests were performed: independent samples t-tests on pre-test scores to ensure group comparability; paired samples t-tests to assess score changes within each group from pre-test to post-test; an independent samples t-test to compare post-test scores between groups; ANCOVA to compare post-test scores while controlling for pre-test scores; and calculations of effect sizes (Cohen's d and partial eta squared) to determine the magnitude of group differences. This comprehensive statistical approach aimed to provide a thorough analysis of the data and evaluate the effectiveness of the intervention.

Results

Pre-test Comparison

Table 1 and Table 2 below present the results of the t test analysis performed on the pretest scores.

Table 1. Descriptive statistics

Group	N	Mean	Std. Deviation (SD)
Experiment	60	52.3	8.7
Control	60	51.8	9.1

Table 2. Independent samples t-test results

t	Df	Sig. (2 tailed) p	Mean difference	95% confidence interval
0.31	118	0.76	0.5	[-1.2, 3.1]

Tables 1 and Table 2 above show descriptive statistics and t test results of the independent samples t-test conducted on the pre-test scores. The results revealed a no significant difference between the experimental group ($M = 52.3$, $SD = 8.7$) and the control group ($M = 51.8$, $SD = 9.1$): $t(118) = 0.31$, $p = 0.76$. This finding indicates that the groups were comparably competent in their knowledge of probability prior to the intervention, allowing any observed differences in post-test results to be attributed solely to the effects of the intervention.

Within-Group Changes

Table 3 below shows how each group changed in performance from pre-test to post-test.

Table 3. Independent samples test

Group	T	df	Sig. (2-tailed)	Effect Size (d)
Experimental	15.62	59	< 0.001	2.02
Control	8.74	59	< 0.001	1.13

The paired samples t-tests in Table 3 above indicated significant improvements from pre-test to post-test for both groups. These results demonstrate significant improvements in both groups, with the experimental group exhibiting a larger effect size ($d = 2.02$) compared to the control group's effect size of ($d=1.13$).

Post-test Comparison

Table 4. Descriptive statistics for post-test scores

Group	N	Mean	Std Deviation
Experimental	60	76.50	10.20
Control	60	68.90	11.70

Table 5. Independent samples t-test results for post-test scores

	t	df	Sig. (2-tailed)	d
Equal variances assumed	3.72	118	< 0.001	0.68

From the data in Tables 4 and 5, the post-test independent samples t-test revealed significantly higher test scores in the experimental group ($M = 76.5$, $SD = 10.2$) compared to the control group ($M = 68.9$, $SD = 11.7$), $t\,(118) = 3.72$, $p < 0.001$, $d = 0.68$.

Post-test Comparison

Table 6. ANCOVA summary

Source	F	df	Sig.	Partial η^2
Intervention	15.23	1.117	< 0.001	0.12

With pre-test scores as a covariate, the results indicate a significant effect of the intervention on post-test scores, $F\,(1, 117) = 15.23$, $p < 0.001$, partial $\eta^2 = 0.12$. This suggests that the argumentation-based approach resulted in higher post-test scores when controlling for initial differences in pre-test performance.

Discussion

The findings of this study support the effectiveness of the argumentation-based instruction in enhancing students' understanding and performance in probability at the advanced-level. This aligns with previous research that has highlighted the positive impact of argumentation on mathematical learning outcomes. For instance, (Dede, 2018) emphasizes the importance of collective argumentation in mathematics education, noting that it supports deeper engagement and understanding among students. Similarly, Walidah (2021) found that argumentation enhances students' abilities to construct and evaluate mathematical proofs, which is crucial for developing a robust understanding of mathematical concepts.

In relation to Hypothesis 1 (H1), the significant difference in post-test scores between the experimental group and the control group ($M = 76.5$ vs. $M = 68.9$) corroborates findings from (Iqbal & Akbar, 2021), who reported substantial improvements in critical thinking skills among prospective teachers when employing argumentation-based strategies. The medium effect size ($d = 0.68$) observed in this study is not directly comparable to the findings of Mohammed (2020), who demonstrated improvements in health-related fitness outcomes in university students through different pedagogical strategies.

Regarding Hypothesis 2 (H2), the ANCOVA results indicated that the argumentation intervention had a significant effect on students' ability to apply probability concepts to real-world problems, with partial $\eta^2 = 0.12$. This finding resonates with the work of (Stylianides & Stylianides, 2017), who highlighted the necessity of research-based interventions in mathematics education to enhance students' problem-solving abilities. The current study's results suggest that argumentation not only aids in theoretical understanding but also equips students with practical skills necessary for real-world applications, echoing the sentiments of (Erduran et al., 2015), who noted the importance of argumentation in developing epistemic practices in science education.

For Hypothesis 3 (H3), the observation that lower-performing students showed the greatest absolute gains in their scores aligns with the findings of (Brown, 2017), who emphasized the potential of argumentation to support equity in mathematics learning. Although the differences in gains among performance levels did not reach statistical significance, the trend suggests that argumentation-based instruction may be particularly beneficial for students who struggle with traditional teaching methods.

The results of this study contribute to the growing body of literature that advocates for the integration of argumentation in mathematics education. The findings extend previous research by demonstrating that argumentation not only enhances students' theoretical understanding of probability but also improves their ability to apply this knowledge in practical situations. This is particularly crucial for advanced-level students, who must navigate complex mathematical concepts and real-world applications. Despite the positive outcomes, it is important to acknowledge the limitations of this study. The research was conducted in a single school in Bulawayo, which limits the generalizability of the findings to other contexts. Future research therefore should aim to include a larger and more diverse sample across different regions of Zimbabwe to validate these results. Additionally, longitudinal studies could provide insights into the long-term effects of argumentation on students' understanding and application of probability.

The current study provides evidence for the effectiveness of the argumentation-based instruction in enhancing students' understanding and performance in probability. The findings underscore the need for educational institutions to incorporate argumentation into their mathematics curricula, as it improves critical thinking and problem-solving skills essential for the success in an increasingly complex, data-driven world. Through aligning pedagogical practices with the principles of argumentation, teachers can better prepare students for the challenges they will face in higher education and beyond.

Conclusion and Implications

The current quantitative study provides compelling evidence regarding the effectiveness of probabilistic argumentation at the advanced-level within a Zimbabwean high school context, enhancing students' understanding and performance. The improved post-test scores and medium effect size for the experimental group indicate that this approach holds significant potential for enhancing students' understanding of probability and critical thinking skills.

Based on the findings, the study recommends that teacher education colleges and higher education institutions offer mathematics teacher trainees an argumentation training course that encompasses both theoretical foundations and practical classroom implementation. The

Ministry of Primary and Secondary Education, along with publishers and textbook writers, should incorporate argumentation activities related to probability topics in mathematics textbooks and other learning materials. These activities should engage students in constructing and critiquing mathematical arguments. Furthermore, to effectively evaluate the gains from argumentation instruction, the Zimbabwe School Examination Board should reform its assessment methods to include questions that assess students' abilities in constructing and analysing probabilistic arguments, thereby aligning assessment with pedagogical practices and enhancing the benefits of argumentation instruction. Long-term studies could then be conducted to evaluate the enduring efficacy of instructional argumentation on students' probabilistic thinking and its application in subsequent mathematics classes and real-life contexts.

The recommendations made in this study aim to improve the quality of probability teaching at the advanced-level within the Zimbabwean education system. This will not only enhance students' chances of excelling academically but also foster the development of critical thinking and reasoning skills, benefiting learners across various disciplines and real-life situations. Evidence-based, innovative argumentation could become a vital critical thinking skill, preparing students for the challenges of higher education in an increasingly complex, data-driven world, and positioning Zimbabwe to excel in education.

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