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Article

Development of an alternative ranked set sampling estimators

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Abstract

Ranked Set Sampling (RSS) is a well-established technique that improves estimation efficiency by incorporating ranking information before selecting a sample. Conventional RSS estimators such as the Mean RSS estimator, is not appropriate when observations are measured in rates or are periodical and in the presence of skewed or heavy-tailed distributions. This study develops credible-alternative Ranked Set Sampling (RSS) estimators, namely the Harmonic Mean RSS (HMRSS), Geometric Mean RSS (GMRSS), and Trimmed Mean RSS (TMRSS) estimators that are robust to data measured in rate, period and less sensitive to extreme values. The proposed RSS estimators were validated with artificial datasets with varying values of σ^2 = $0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0$, $k = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9$ and $\alpha = 0.05, 0.10, 0.15, 0.20, 0.25, 0.30, 0.35, 0.40, 0.45$. Percentage relative efficiency, $PRE = [\frac{Var(existing\ estimator)}{Var(proposed\ estimator)} \times 100]\%$ was used as a criterion to judge the efficiency of the proposed estimators against the orthodox estimators. A $PRE >$ 100% indicates efficiency of the proposed estimator over the existing ones. Variances of MRSS, HMRSS, GMRSS and TMRSS were 0.0333, 0.00078, 0.0033, and 0.0370 respectively, when $n = 5, m = 3, k = 0.1$, $\alpha = 0.05$ and $\sigma^2 = 0.5$. Results when $n = 10, m = 5, k = 0.3, \alpha = 0.10$ and $\sigma^2 = 1.0$ were $0.0333, 0.0005, 0.0060$ and 0.0250 respectively, and $0.0286, 0.0004, 0.0071$ and 0.0204 for $n = 15$, $m = 7, k = 0.5, \alpha = 0.15$ and $\sigma^2 = 1.5$. The results indicate that both HMRSS and GMRSS outperform the orthodox MRSS in terms of efficiency, particularly when dealing with skewed or heavy-tailed distributions. However, the TMRSS estimator, despite its robustness against outliers, showed mixed performance and less efficient to MRSS estimator.

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Ranked set sampling; estimator, efficiency; percentage relative efficiency; heavy-tailed distributions

Introduction

Ranked set sampling (RSS), invented by McIntyre (1952), improves the efficiency of estimators by incorporating ranking information prior selecting a sample. RSS is efficient (lower variance), cost-effective (smaller sample size needed for higher level of precision), robust (less sensitive to outliers and skewed data) and has wider scope (suitable for both finite and infinite populations) when compared to simple random sampling (SRS). RSS emerged as a remedy to challenges in agricultural research, where the cost of measuring crop yield or soil quality is exorbitant or time-intensive. Takahasi & Wakimoto (1968), Dell and Clutter (1972) are early works on RSS. Notable interventions include and not limited to: Chen et al., 2003; Perron &

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Sinha 2004; Jemain et al., 2008; Munir et al., 2010; Wolfe, 2012; Al-Omari & Raqab 2013). Recent studies extend RSS to more complex sampling strategies: Extreme RSS that selects extreme values within ranked sets which is useful for detecting rare events or anomalies, Median RSS that reduces bias in skewed distributions, Generalized RSS handling multivariate data and complex ranking schemes. Effectiveness of RSS has also been demonstrated by several applications and simulation studies across diverse disciplines: Computational statistics (Sevinc et al., 2017). Environmental sciences to monitor pollutant levels in air, water, or soil and estimating species diversity or biomass in ecological studies with reduced measurement costs (Kaur et al., 2010). Medical sciences and public health to improve the efficiency of diagnostic tests by ranking patients based on clinical indicators and estimating the prevalence of rare diseases or anomalies in large populations. Physical sciences and Engineering as industrial quality control measure to evaluate product defects or reliability in manufacturing processes (Al-Omari & Ibrahim, 2020). Agriculture sciences to estimate crop yields or soil properties using visual ranking or expert judgment and reducing the cost of destructive sampling in testing grain quality or nutrient content (Arzu and Derya, 2020). Economics and social sciences survey using income level, educational attainment as auxiliary ranking variable (Hakan & Ozçomak, 2024).

In traditional SRS, n from N units are randomly drawn with or without replacement and all selected units are measured. In contrast, RSS selects n^2 from N and the selected subjects are partition into k sets of equal size n, units in each set are then ranked based on some criterion or judgment without considering actual measurement. Unit y_{11} is drawn from first group or set, y_{22} from the second set, y_{33} from the third group and so on until y_{kk} is selected from k^{th} set. The reason for the acronym ranked set sampling. While RSS has shown significant potential in improving the efficiency of sampling designs, conventional Mean Ranked Set Sampling (MRSS) estimator propounded by McIntyre (1952), and its mathematical framework developed by Takahasi & Wakimoto (1968) as is not appropriate (may produce biased or inefficient estimates) when observations are measured in rates (e.g., inflation) or are periodical (time dependent) or the sampled population is heavy-tail. This limitation underscores the need for the development of alternative RSS estimator that is robust to ranking errors and applicable across a wider range of measuring scenarios. Thus, the study develops alternative (namely: Harmonic, Geometric and Trimmed mean) RSS estimators, derive their sampling distributions and explore efficiency of the proposed estimators with frontier estimator.

$$MRSS = \frac{1}{n}\sum_{i=1}^{n} Y_{i(i)} \qquad and \qquad Var(MRSS) = \frac{\sigma^2}{nm} \tag{1}$$

Method

Arithmetic mean is the most commonly used estimator of central tendency, while the arithmetic mean is easy to compute and interpret, it is sensitive to outliers and skewed distributions, which can lead to a biased estimate. Harmonic, Geometric and Trimmed mean are other credible alternatives depending on the nature of the data. However, their performance in the context of ranked set sampling framework has not been explored, which motivates the current study.

Harmonic Mean Ranked Set Sampling Estimator (HMRSS)

If $x_1, x_2, \cdots, x_n$ is a set of observations, then Harmonic Mean (HM) is defined as the reciprocal of the arithmetic mean of the reciprocals of the observations. Mathematically,

$$HM = \frac{n}{\sum_{i=1}^{n} \frac{1}{x_i}} \tag{2}$$

Equation (2) is a non-linear function of random variables X_{ij} then Taylor's series expansion is used to approximate the mean and variance. Let $T = \frac{1}{n}\sum_{i=1}^{n} \frac{1}{x_i}$, therefore,

$$HM = g(T) = \frac{1}{T} \qquad g'(T) = -\frac{1}{T^2} \qquad and \qquad g''(T) = \frac{2}{T^3} \tag{3}$$

Needs to be approximated so that

$$g(T) \approx \frac{1}{\mu T} - \frac{1}{\mu^2 T}(T - \mu T) + \frac{1}{\mu^3 T}(T\mu T)^2 \tag{4}$$

Since $T = \frac{1}{n}\sum_{i=1}^{n} \frac{1}{x}$ and if X has a mean μ, $E(T)$ is derived as

$$E(T) = \frac{1}{n}\sum_{i=1}^{n} E\left(\frac{1}{Xi}\right) = E\left(\frac{1}{X}\right) \approx \frac{1}{\mu} + \frac{Var(X)}{\mu^3} = \frac{1}{\mu} + \frac{\sigma^2}{\mu^3} = \mu T \tag{5}$$

And for large n, the variance of T is

$$Var(T) = \frac{1}{n} Var\left(\frac{1}{X}\right) \approx \frac{1}{n}\frac{\sigma^2}{\mu^4} = \frac{\sigma^2}{n\mu^4} \tag{6}$$

$$E(HM) = E[g(T)] \approx \frac{1}{\mu T} + \frac{1}{\mu^3 T} Var(T) = \mu - \frac{\sigma^2}{\mu^3} \tag{7}$$

To incorporate this into the RSS framework, weights W_i ($\sum_{i=1}^{M} W_i = 1$), are needed such that

$$HMRSS = \sum_{i=1}^{m} W_i g(T) = \sum_{i=1}^{m} W_i \frac{n}{\sum_{i=1}^{n} \frac{1}{xi}} \tag{8}$$

And therefore

$$E(HMRSS) = \sum_{\{i=1\}}^{\{M\}} W_i \left(\mu - \frac{\sigma^2}{\mu^3}\right) = \left(\mu - \frac{\sigma^2}{\mu^3}\right) \sum_{i=1}^{M} W_i = \mu - \frac{\sigma^2}{\mu^3} \tag{9}$$

With variance

$$Var[g(T)] \approx [g'(\mu T)]^2 Var(T) = \left[-\frac{1}{T^2}\right]^2 Var(T) = \frac{1}{\mu^4} Var(T) \approx \frac{\left(1 + \frac{\sigma^2}{\mu^2}\right)^2 \sigma^2}{n\mu^4} \tag{10}$$

Geometric Mean Ranked Set Sampling Estimator (GMRSS)

The geometric mean of a set of numbers $x_1, x_2, \cdots, x_n$ is the nth root of the product of the observations, thus $\sum_{i=1}^{n} x_i$

$$GM = \begin{cases} \sqrt[n]{x_1 . x_2 \cdots x_n}, \text{ for ungrouped data} \\ \sqrt[\sum_{i=1}^{n} x_i]{x_1^{f_1} . x_2^{f_2} . \cdots . x_n^{f_n}}, \quad \text{for grouped data} \end{cases} \tag{11}$$

The Geometric mean ranked set sampling (GMRSS) estimator utilizes the geometric mean of ranked units and is defined as :

$$GMRSSS = \prod_{i=1}^{n} (X_t)^{\frac{1}{n}} = \exp[\tfrac{1}{m} \textstyle\sum_{i=1}^{m} In(X_t)] \tag{12}$$

Where m is the set size, X_t denotes the nth order statistics of the RSS process. The mean of (12) is

$$\mathrm{E}[GMRSSS] = \exp[\tfrac{1}{m} \textstyle\sum_{i=1}^{m} E[In(X_t)]]$$

$$= exp \ \frac{1}{m} \sum_{i=1}^{m} E\{In(\mu) + In\left(1 + \frac{X_t - \mu}{\mu}\right)\}]$$

$$\approx \exp \frac{1}{m} \sum_{i=1}^{m} E\{In(\mu) + \frac{X_t - \mu}{\mu} - \frac{(X_t - \mu)\hat{}2}{2\mu^2}\}]$$

$$\approx \exp\left[In(\mu) - \frac{\sigma^2}{2\mu^2}\right]; \qquad since \quad E[X(t)] = \mu \qquad and \qquad Var(X(t) = \sigma^2$$

$$\approx \mu\left(1 - \frac{\sigma^2}{2m\mu^2}\right) \tag{13}$$

For large m, the second term in (13) vanishes, giving E[μGMRSS] ≈ μ. Thus, the GMRSS estimator is asymptotically unbiased with

$$Var(\mathrm{GMRSS}) \approx \mu^2 \left(1 - \frac{\sigma^2}{m\mu^2}\right) - \mu^2 \left(1 - \frac{\sigma^2}{2m\mu^2}\right)^2 \approx \frac{k\sigma^2}{nm} \tag{14}$$

Where k is a constant depending on the RSS procedure.

Trimmed Mean Ranked Set Sampling Estimator (TMRSS)

Let $x_1, x_2, \cdots, x_n$. The trimmed mean of X is defined as:

$$TM = \frac{1}{n - 2k} \sum_{i=k+1}^{n-2k} X_i \tag{15}$$

Where; n = total no of observations, $k = \sigma n$ number of trimmed values, X_i = the remaining values in the trimmed dataset. The trimmed-Mean RSS estimator,TMRSS, is

$$TMRSS = \sum_{i=1}^{m} w_i T_i = \sum_{i=1}^{m} w_i \frac{1}{n - 2k} \sum_{i=k+1}^{n-2k} X_t \tag{16}$$

Such that $\sum_{i=1}^{m} W_i = 1$. The estimator is unbiased as

$$E(TMRSS) = E[\sum_{i=1}^{m} w_i T_i] = \sum_{i=1}^{m} w_i E(T_i) = \mu$$

and

$$Var(T) = \frac{1}{(n - 2k)^2} \sum_{i=k+1}^{n-2k} Var(X_i) \approx \frac{(1 - 2\alpha)\sigma^2}{n(1 - 2\alpha)^2} = \frac{\sigma^2}{n(1 - 2\alpha)} \tag{17}$$

So that

$$Var(TMRSS) = Var\left[\sum_{i=1}^{m} W_i T_i\right] = \sum_{i=1}^{m} W_i^2 Var(T_i) = \frac{\sigma^2}{n(1-2\alpha)} \sum_{i=1}^{m} W_i \quad (18)$$

When the weights are uniform, $W_i = \frac{1}{m}$, then

$$Var(TMRSS) = \frac{\sigma^2}{nm(1-2\alpha)} \quad (19)$$

The Percent Relative Efficiency (PRE) is given as

$$PRE(\hat{\theta}_E, \hat{\theta}_P) = \frac{Var(\hat{\theta}_E)}{Var(\hat{\theta}_P)} X\ 100 \quad (20)$$

where θ_P denotes the proposed estimator (HMRSS, GMRSS, or TMRSS), θ_E represents the baseline MRSS estimator.

Findings

This section presents results and a discussion of the findings by using an artificial dataset of varying sample size n, mean μ , variance σ^2, scaling factor k, number of cycles m, and trimming proportion α to validate the efficiency of the proposed estimators.

Harmonic Mean RSS Estimator

Using (1), (10) and (20), the values for the variances of the MRSS, HMRSS estimators and the Percent Relative Efficiency of the two are calculated and the results presented in Table 1.

Table 1. Performance of HMRSS against the conventional MRSS

n	m	μ	σ^2	V(HMRSS)	V(MRSS)	PRE (%)
5	3	5	0.5	0.00078	0.03333	4273.1
			1	0.00157	0.06667	4329.2
			1.5	0.00226	0.1	4447.8
			2	0.00296	0.1333	4504.7
			2.5	0.00329	0.16667	4578.8
			3	0.00429	0.2	4626.3
			3.5	0.00552	0.26667	4780.9
			4	0.00619	0.3	4872.1
10	5	6	0.5	0.00023	0.017	4383.8
			1	0.00045	0.03333	4338.8
			1.5	0.00088	0.05	4444.4
			2	0.00108	0.08333	4577.6
			2.5	0.00148	0.1	4708.6
			3	0.00167	0.13333	4786.4
			3.5	0.00172	0.16667	4864.9
			4	0.00185	0.18334	4893.2

15	*7*	*7*	*0.5*	*0.00019*	*0.00952*	*5148.6*
			1	*0.00028*	*0.019*	*5167*
			1.5	*0.00037*	*0.02857*	*5194.5*
			2	*0.00046*	*0.03333*	*5290.5*
			2.5	*0.00055*	*0.0381*	*5294.7*
			3	*0.00072*	*0.04286*	*5352.5*
			3.5	*0.00081*	*0.04467*	*5372.1*
			4	*0.00087*	*0.04734*	*5536.4*
20	*9*	*8*	*0.5*	*0.00005*	*0.00278*	*5660*
			1	*0.00009*	*0.00556*	*6172.8*
			1.5	*0.00017*	*0.00833*	*5795*
			2	*0.00019*	*0.01389*	*5847.6*
			2.5	*0.00028*	*0.01667*	*6391.1*
			3	*0.00032*	*0.01944*	*5935.5*
			3.5	*0.00037*	*0.02222*	*6075*
			4	*0.00041*	*0.025*	*6097.6*

Geometric Mean RSS Estimator

Using (1), (14) and (20), the results for the variances of the MRSS, GMRSS estimators and the PRE of GMRSS to MRSS are calculated respectively.

Table 2. Performance of GMRSS estimator against the conventional MRSS

n	m	σ^2	k	V(GMRSS)	V(MRSS)	PRE (%)
5	*3*	*0.5*	*0.1*	*0.0033*	*0.0333*	*1009.09*
			0.3	*0.01*	*0.0333*	*333*
			0.5	*0.0167*	*0.0333*	*199.4*
			0.7	*0.0233*	*0.0333*	*142.92*
			0.9	*0.03*	*0.0333*	*111*
10	*5*	*1*	*0.1*	*0.002*	*0.02*	*1000*
			0.3	*0.006*	*0.02*	*333.33*
			0.5	*0.01*	*0.02*	*200*
			0.7	*0.014*	*0.02*	*142.86*
			0.9	*0.018*	*0.02*	*111.11*
15	*7*	*1.5*	*0.1*	*0.0014*	*0.0143*	*1021.43*
			0.3	*0.0043*	*0.0143*	*332.56*
			0.5	*0.0071*	*0.0143*	*201.41*
			0.7	*0.01*	*0.0143*	*143*
			0.9	*0.0129*	*0.0143*	*110.85*
20	*9*	*2*	*0.1*	*0.0011*	*0.0111*	*1009.09*
			0.3	*0.0033*	*0.0111*	*336.36*

			0.5	0.0056	0.0111	198.21
			0.7	0.0078	0.0111	142.31
			0.9	0.01	0.0111	111
25	11	2.5	0.1	0.0009	0.0091	1011.11
			0.3	0.0027	0.0091	337.04
			0.5	0.0045	0.0091	202.22
			0.7	0.0064	0.0091	142.19
			0.9	0.0082	0.0091	110.98
30	13	3	0.1	0.0007	0.0077	1100
			0.3	0.0023	0.0077	334.78
			0.5	0.0038	0.0077	202.63
			0.7	0.0054	0.0077	142.59
			0.9	0.0069	0.0077	111.59
35	15	3.5	0.1	0.0006	0.0067	957.14
			0.3	0.002	0.0067	335
			0.5	0.0033	0.0067	203.03
			0.7	0.0047	0.0067	142.55
			0.9	0.006	0.0067	111.67

Trimmed Mean RSS Estimator

Using (1), (19) and (20), the values for the variances of the MRSS, TMRSS estimators and their PRE are calculated and the results are shown.

Table 3. Performance of TMRSS estimator against the conventional MRSS

n	m	σ^2	α	Var(TMRSS)	Var(MRSS)	RE
5	3	0.5	0.05	0.037	0.0333	0.9
10	5	1	0.1	0.025	0.02	0.8
15	7	1.5	0.15	0.0204	0.0143	0.7
20	9	2	0.2	0.0185	0.0111	0.6
25	11	2.5	0.25	0.0182	0.0091	0.5
30	13	3	0.3	0.0192	0.0077	0.4
35	15	3.5	0.35	0.0222	0.0067	0.3
40	17	4	0.4	0.0294	0.0059	0.2
45	19	4.5	0.45	0.0526	0.0053	0.1

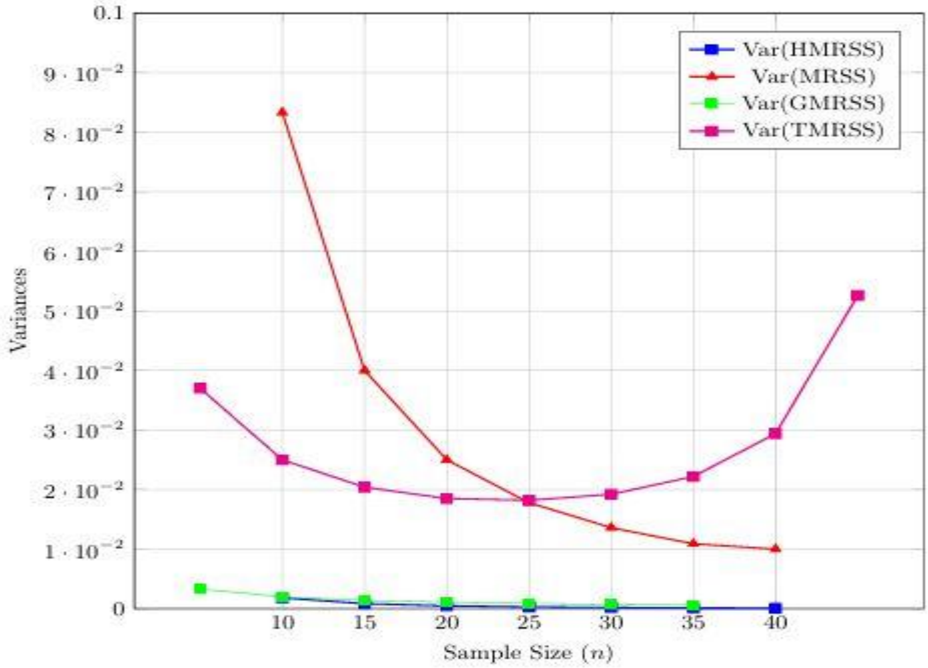

Figure 1. Efficiency plots of the HMRSS, GMRSS and TMRSS estimators for varying values of n and m when $k = 0.30, \alpha = 0.05, \mu = 5.00$ and $\sigma^2 = 0.50$

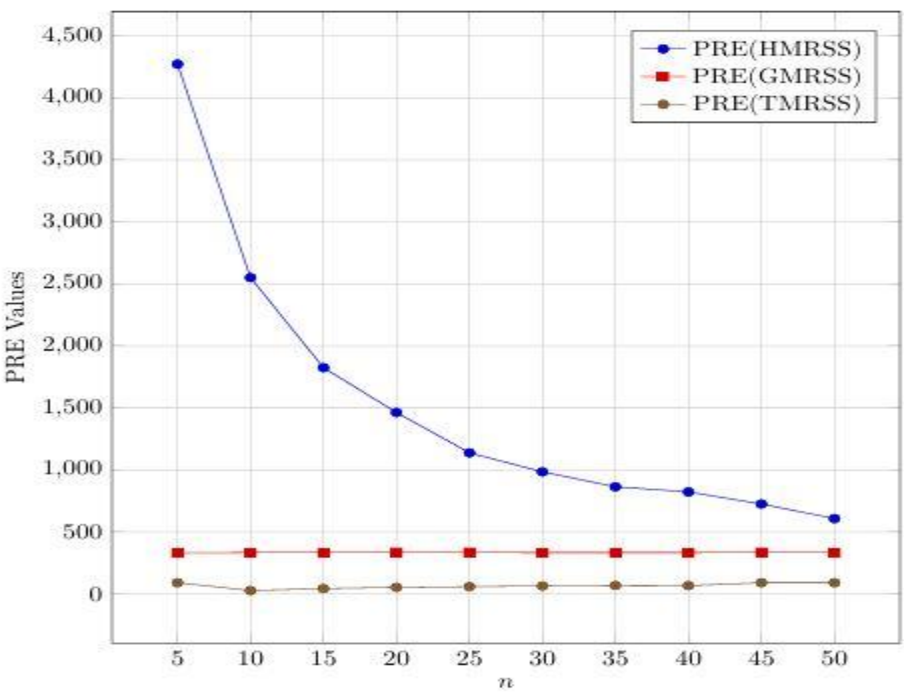

Figure 2. PRE plots of the HMRSS, GMRSS and TMRSS estimators for varying values of n and m when $k = 0.30, \alpha = 0.05, \mu = 5.00$ and $\sigma^2 = 0.50$

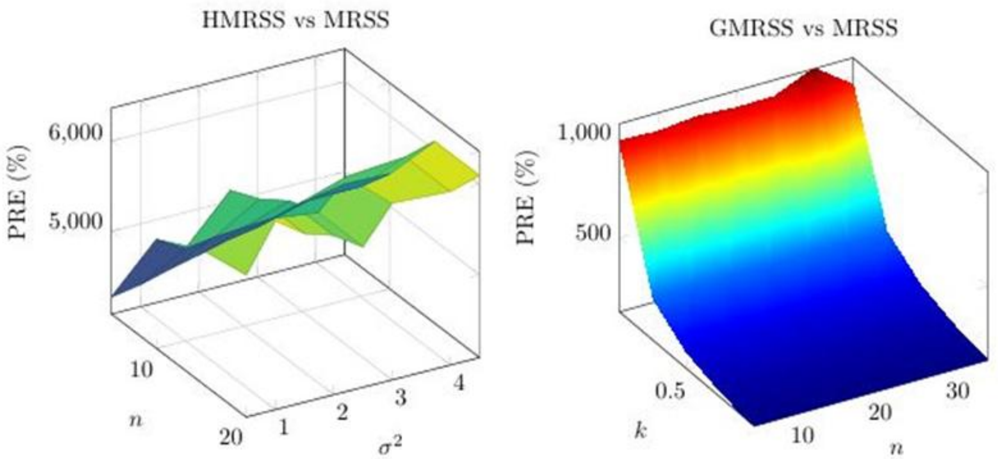

Figure 3. 3D PRE Plots of HMRSS and GMRSS vs MRSS for varying values of σ^2, k and n

Discussion

Evidence from Table 1 and Figure 1, HMRSS consistently yields lower variance than MRSS, indicating higher precision. As sample size (n) increases, both variances decrease, but HMRSS declines faster. The highest gain in efficiency occurs at small sample sizes, making HMRSS ideal for limited data scenarios. Similarly, Figure 3 depicts that PRE values remain high across all sample sizes, indicating that HMRSS and GMRSS consistently outperform MRSS. At smaller sample sizes ($n = 5, 10, 15$), PRE is significantly higher ($> 100\%$), showing drastic efficiency improvements. Beyond $n = 30$, though GMRSS remains superior but PRE values show a diminishing trend.

Table 2 and Figure 1 depict that GMRSS is highly efficient compared to MRSS, especially when sample sizes are small. As population variance (σ^2) increases, the efficiency of GMRSS remains high but with slightly diminishing gains. GMRSS is recommended for applications requiring high-precision estimations with limited samples. These findings confirm the theoretical advantage of GMRSS in practical sampling applications, reinforcing its use over MRSS when sample size is small. statistical estimations.

Based on the results in Table 3, the Var(MRSS) decreases significantly as n increases, demonstrating improved precision with larger datasets. Var(TMRSS) reduces as n increases, it slightly stabilized between $20 \leq n \leq 25$. Beyond $n = 25$, Var(TMRSS) grows again. The relative efficiency decreases linearly from 0.90 to 0.10 as n increases from 5 to 45. This suggests that TMRSS is more efficient for smaller sample sizes but not for larger ones. That is, TMRSS is relatively efficient when the sample size is small ($\leq$ 15). For larger n, it becomes less efficient compared to MRSS. Parameters such as σ^2 and α affect the variance for TMRSS more significantly compared to MRSS, underscoring the need for careful parameter selection during the design phase.

Using Figure 2 and Figure 3, the HMRSS and GMRSS have highest PRE values across varying values of n. The PRE values for HMRSS, GMRSS and TMRSS decrease steadily as n, k, σ^2 increases, affirming that RSS designs are valid with small sample size, hence minimizing the survey cost.

Conclusion and Implications

The findings from this research provide strong evidence that alternative RSS estimators such as HMRSS and GMRSS are viable and superior alternatives to the extant MRSS estimator evidenced from reduced variance and increased percent relative efficiency irrespective of values of n, m, σ^2, k making them to performed exceptionally well, demonstrating superior efficiency and serving as credible alternatives in practical applications where higher precision of population parameters is demanded. Conversely, the TMRSS estimator outperforms the MRSS estimator on very few occasions, which suggests that TMRSS may be better suited for specific data scenarios where extreme observations contribute excessive noise rather than providing useful information. Overall, the study infers that both HMRSS and GMRSS estimators significantly enhance efficiency and reliability in parameter estimation, offering more informative statistical inference than the traditional MRSS method.

Arising from the findings and conclusion above, the following recommendations have been advanced: The HMRSS and GMRSS estimators should be considered for application in real-world problems. Future studies should explore conditions under which the Trimmed Mean RSS estimator may outperform MRSS. By implementing these recommendations, statisticians can maximize the benefits of the proposed alternative RSS estimators, particularly HMRSS and GMRSS in any real applications.

Declarations

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